Stabilizing E-scooters at Zero Speed with Feedback Control ⋆

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Abstract: The dynamics of electric scooters is analyzed via a spatial mechanical model. The e-scooter is balanced at zero speed, by applying internal driving torque to the front wheel. A hierarchical, linear state feedback controller is designed with feedback delay. The linear stability charts of the delayed controller are constructed with semi-discretization. The effects of the feedback delay and the center of gravity position of the handlebar on the linear stability are investigated. The performance of the control algorithm is tested by numerical simulations with the nonlinear governing equations.

Keywords: Electric scooter, Nonlinear dynamics, Stability, Control of nonlinear systems, Feedback delay, Numerical simulation

1. INTRODUCTION

Since micromobility vehicles such as electric scooters (e-scooters) and unicycles provide a great solution for the ,,first and last mile problem", they are very popular nowadays in road transportation. Unfortunately, e-scooters are often scattered in the streets, since many users leave them in inappropriate places, e.g. in the middle of the walkways. As a futuristic solution, we propose that e-scooters could drive themselves to docking stations or designated parking areas. In this study, we analyze the stability of riderless e-scooters, which can balance themselves in the vertical position.

2. MECHANICAL MODEL

The investigated spatial mechanical model, which is based on the Whipple bicycle model (Whipple, 1899), can be seen in Fig. 1. The multibody system consists of four rigid bodies: the handlebar and fork assembly, the body (the frame), and the front and rear wheels.

The nonlinear equations of motion can be derived considering the geometric and kinematic constraints of the system, e.g., with the help of Kane's method (Kane and Levinson, 1985). In order to stabilize the e-scooter at zero speed, we turn the front wheel into a $\delta = \pi/2$ steering angle position, and we apply internal driving torque M^{d} on the front wheel.

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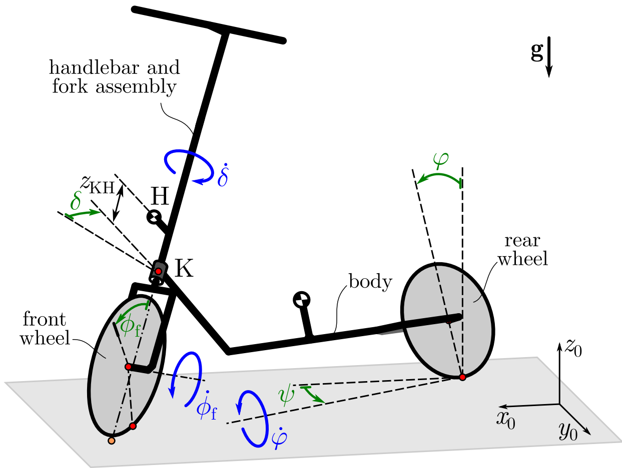

Fig. 1. The spatial mechanical model of an electric scooter

In that configuration, the small motions around the upright position ($\varphi = 0$) can be described by the linearized equation of motion $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{Q}$, where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix and $\mathbf{Q} = \left[0 \ M^{\mathrm{d}}\right]^{\mathrm{T}}$ is the vector of generalized forces with internal driving torque M^{d}. The vector of state variables $\mathbf{x} = [\varphi \ \phi_{\mathrm{f}}]^{\mathrm{T}}$ contains the lean angle φ and the front wheel angle ϕ_{f}. The position z_{KH} of the center of gravity of the handlebar H (see Fig. 1) has a crucial role in terms of the stability of the e-scooter.

3. CONTROL DESIGN AND RESULTS

We design a hierarchical, linear state feedback controller. Namely, a higher-level controller calculates the desired front wheel angle as

$$\phi_{\mathrm{f,des}} = -K^{\mathrm{d}}_{\mathrm{p}\varphi}\varphi(t-\tau) - K^{\mathrm{d}}_{\mathrm{d}\varphi}\dot{\varphi}(t-\tau)\,, \qquad (1)$$

where $K^{\mathrm{d}}_{\mathrm{p}\varphi}$ and $K^{\mathrm{d}}_{\mathrm{d}\varphi}$ are proportional gains for the lean angle and lean rate, respectively; and τ is the feedback delay in the controller. The internal driving torque M^{d} is created by a lower-level control as

$$M^{\mathrm{d}} = -K^{\mathrm{d}}_{\mathrm{p}\phi_{\mathrm{f}}} \left(\phi_{\mathrm{f}}(t) - \phi_{\mathrm{f,des}}\right) - K^{\mathrm{d}}_{\mathrm{d}\phi_{\mathrm{f}}} \dot{\phi}_{\mathrm{f}}(t)\,, \qquad (2)$$

where $K^{\mathrm{d}}_{\mathrm{p}\phi_{\mathrm{f}}}$ and $K^{\mathrm{d}}_{\mathrm{d}\phi_{\mathrm{f}}}$ are proportional gains for the front wheel angle and front wheel rate, respectively. We assume that the time delay in the lower lever controller is negligible compared to the time delay in the higher-level controller.

To analyze the linear stability properties of the delayed controller, we construct linear stability charts with the help of semi-discretization (Insperger and Stepan, 2011). In this study, the lower-level control gains are fixed, i.e., $K^{\mathrm{d}}_{\mathrm{p}\phi_{\mathrm{f}}} = -145\,\mathrm{Nm}$ and $K^{\mathrm{d}}_{\mathrm{d}\phi_{\mathrm{f}}} = -30\,\mathrm{Nms}$ and geometric parameters are based on an existing e-scooter (Klinger et al., 2022). The stability boundaries are shown in the plane of the higher-level control gains $K^{\mathrm{d}}_{\mathrm{p}\phi_{\mathrm{f}}}$ and $K^{\mathrm{d}}_{\mathrm{d}\phi_{\mathrm{f}}}$ in Fig. 2. The white and gray areas correspond to linearly unstable and linearly stable domains, respectively. In panels (a) and (b), the different shades of gray correspond to different values of the feedback delay τ and the center of gravity position z_{KH}, respectively.

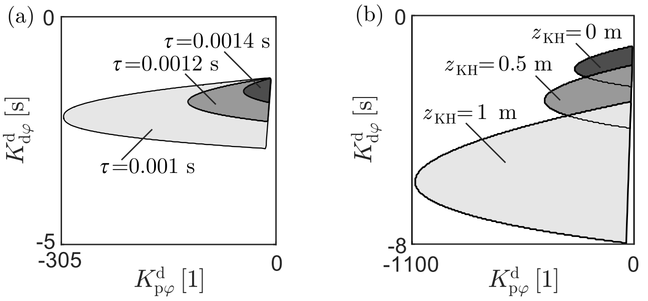

Fig. 2. Linear stability charts of the delayed controller: (a) the effect of the feedback delay τ for $z_{\mathrm{KH}} = 0.27\,\mathrm{m}$, (b) the effect of the handlebar's center of gravity position z_{KH} for $\tau = 0.001\,\mathrm{s}$

As can be observed in Fig. 2(a), the larger the feedback delay value is, the smaller the stable region is. As the stability charts of Fig. 2(b) suggest, the stable region is larger for larger values of z_{KH}. The analysis of human balancing provides similar results and higher center of gravity positions enable larger feedback delays.

We verify the performance of the linear controller with numerical simulations carried out with the nonlinear governing equations. Explicit Euler method was used with a fixed time step of $5\cdot10^{-4}\,\mathrm{s}$ and an impact-like perturbation of the lean rate $\dot{\varphi}(0)$ was applied as an initial condition. Since the steering angle is not prescribed in our general model by any constraint, an internal steering torque

$$M^{\mathrm{s}} = -K^{\mathrm{d}}_{\mathrm{p}\delta}(\delta(t) - \pi/2) - K^{\mathrm{d}}_{\mathrm{d}\delta}\dot{\delta}(t) \qquad (3)$$

was applied to the handlebar to hold $\delta(t) = \pi/2$ and $\dot{\delta}(t) = 0$.

Four of the control gains were fixed, namely $K^{\mathrm{d}}_{\mathrm{p}\phi_{\mathrm{f}}} = -145\,\mathrm{Nm}$, $K^{\mathrm{d}}_{\mathrm{d}\phi_{\mathrm{f}}} = -30\,\mathrm{Nms}$, $K^{\mathrm{d}}_{\mathrm{p}\delta} = 10\,\mathrm{Nm}$ and $K^{\mathrm{d}}_{\mathrm{d}\delta} = 1\,\mathrm{Nms}$ were chosen. The optimal higher-level control gains corresponding to the fastest decay of the vibrations were obtained by semi-discretization for $\tau = 0.001\,\mathrm{s}$. A critical initial lean rate $\dot{\varphi}(0) = 0.2927\,\mathrm{rad/s}$ was identified for which the amplitudes of the vibrations remain the same for a sufficiently long time. This predicts the presence of an unstable limit cycle that bounds the region of attraction of the stable equilibrium. Simulations were run for smaller and larger initial lean rates, namely for $\dot{\varphi}(0) = 0.2500\,\mathrm{rad/s}$ (case A) and for $\dot{\varphi}(0) = 0.2929\,\mathrm{rad/s}$ (case B).

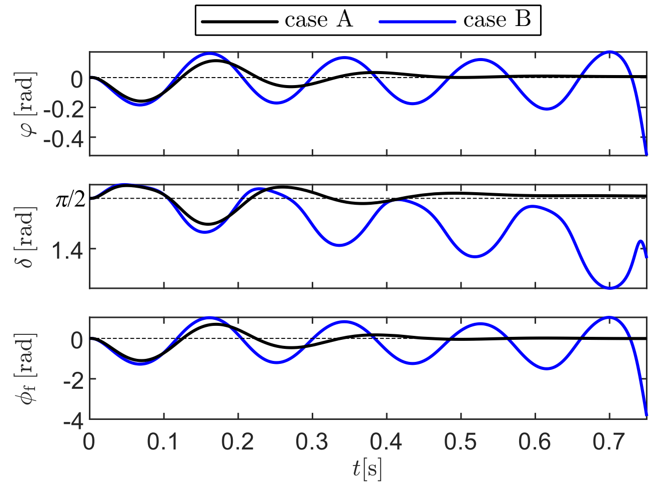

Fig. 3. Nonlinear simulation results using the optimal higher-level control gains ($K^{\mathrm{d}}_{\mathrm{p}\varphi} = -7.99$, $K^{\mathrm{d}}_{\mathrm{d}\varphi} = -1.39\,\mathrm{s}$) for $\tau = 0.001\,\mathrm{s}$

Figure 3 shows the time graphs of the lean angle φ, the steering angle δ and the front wheel angle ϕ_{f}. The black curves correspond to case A, when the initial lean rate was smaller than the critical one. As can be observed, the vibrations decay with time. With the selected control gains, the controller is not able to stabilize the e-scooter for a larger initial lean rate (case B), see the blue curves.

4. CONCLUSIONS

Based on linear stability analysis and numerical simulations, the balancing task of e-scooters can be accomplished for small feedback delay values. However, it can be concluded, that the basin of attraction is small. In the future, the nonlinear analysis and the experimental validation of the theoretical results should be performed.

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